THE DYNAMICAL PROCESS OF A CORONAL TRANSIENT ASSOCIATED WITH AN ERUPTIVE PROMINENCE

II. *Analytical Solutions in Finite Regions*

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(Received 5 January, 1983)

Abstract. Based on the Paper I (Hu, 1983a), the piston model suggested for the coronal transient. The piston may consist of the eruptive prominence and the preceding compressed magnetic field region, and the moving piston is considered to be the driving source of the coronal transient. There is the compressed flow region ahead the piston and the rarefaction region behind the piston. In this paper, two analytical solutions are discussed in finite regions of piston model.

A consistent solution of the ideal magnetohydrodynamical equations including the gravity is obtained for the two-dimensional unsteady configuration in which density and magnetic field are time dependent. Gravity, Lorentz and inertial forces are balanced by the pressure gradient. The analysis gives a consistent radial velocity $u(t)$ which agrees with the typical velocity of eruptive prominence. The density profile is proportional to $1/r$ and propagates outward with the velocity u. This solution may be applied to the piston region of moving plasma and magnetic field.

Furthermore, the compressed flow driven by the piston is discussed. The consistent solution of gas-dynamical equations including solar gravity is obtained for the unsteady and two-dimensional configuration, which is applied to the region between the piston and shock wave. This solution may satisfy the jump conditions of shock wave, which separates the region of compressed flow and quiet corona.

1. Introduction

A coronal transient is considered to be the motion of dense plasma from low corona to the outer corona (see, MacQueen *et al.*, 1974). Many theoretical explanations have been suggested (see, for examples, Rust *et al.*, 1979; MacQueen, 1980). There are three main approaches: namely, the numerical approach (Nakagawa *et al.*, 1978; Wu *et al.*, 1978; Dryer *et al.*, 199), local analysis (Anzer, 1978; Mouschovias and Poland, 1978; Pneuman, 1980; Hu, 1982a) and analytical solutions for infinite space (Low *et al.*, 1982; Low, 1982) and for limited region (Yeh and Dryer, 1981; Hu, 1982b, 1983a, b, c). Theoretical ideas describe many possible mechanisms which may explain some of the transient features. The association of coronal transient with eruptive prominence or with solar flare is analyzed especially by Pneuman (1980) or Dryer *et al.* (1979).

In Paper I (Hu, 1983a), we discussed the gasdynamical effects driven by the piston of moving plasma which consists of eruptive prominence and the preceding compressed magnetic region. Based on the idea suggested in Paper I, the bright feature of the transient corresponds to the compressed plasma ahead the compressed magnetic field region, and not the magnetic field region itself; the dark feature of the transient corresponds to the rarefaction flow behind the piston. The driving force may be the kinetic energy of eruptive prominence or the Lorentz force acting in the compressed

Astrophysics and Space Science **92** (1983) 395–415. 0004–640X/83/0922–0395$03.15.

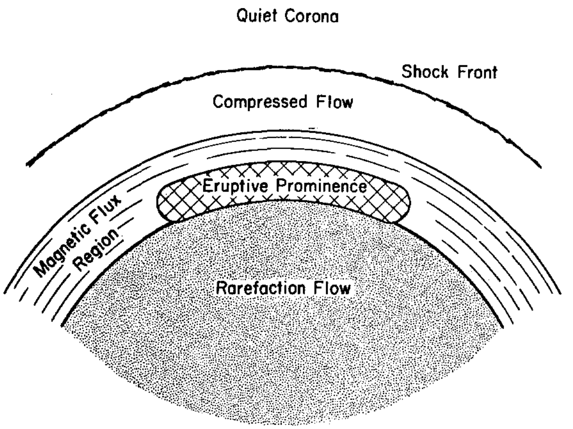

Fig. 1. Schematic of transient process associated with eruptive prominence.

magnetic region, or both. The sketch of the piston model is given in Figure 1. Because the importance of the driving source of the transient, we will analyze the dynamical process of the magnetic field region first. The unsteady and two-dimensional solution of magnetohydrodynamical equations including solar gravity is obtained. This analytical solution is applied to the magnetic region, and is used to discuss the driving mechanism and propagation of the coronal transient. Furthermore, the two-dimensional gas-dynamical equations are solved, and the solution is applied to the region of compressed flow, where the shock wave conditions may be satisfied.

In the next three sections, we discuss the solutions and their properties in the region of piston. The assumptions of the model are described mathematically in Section 2, and the solution of MHD equations is given in Section 3. In Section 4, the solution is applied to the piston region of the coronal transient to discuss the driven mechanism especially. The solution of gasdynamical equations for compressed flow is given in Section 5, and the jump conditions of the solution at shock wave are analyzed in Section 6. One example of the results is applied to the compressed region of piston model. The last section is the discussion, which show the connections between the topography of transient and the configurations of the magnetic field.

2. Assumptions and Equations

The flow field of the transient process may be divided into several typical regions as shown in Figure 1. At first, we discuss the region of compressed magnetic flux which is driven by either the prominence or the Lorentz force in the magnetic region. The boundaries of the magnetic region contacted with the compressed region and the rarefaction region are tangential discontinuous surface. In general, the density in the region of strong magnetic region is lower than outside. As the driving force relates closely with the magnetic region, and as both the bright and dark features of the transient are the gasdynamical response of the magnetic region, it is important to study the dynamical processes in this region.

The magnetohydrodynamical process couples the moving plasma and the magnetic field in the corona. Generally, the unsteady magnetohydrodynamical equations are hyperbolic-elliptic equations (see, for example, Hu, 1977). This means that we will search for the solution which satisfies both the property of propagation and the equilibrium conditions. This is one of the difficulties. Solar gravity has the influence which cannot be neglected, this is another difficulty. Furthermore, we must search for a mathematical solution which agrees with the observed features of the coronal transient.

The unsteady magnetohydrodynamical equations may be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 , \tag{2.1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\nabla p + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \mathbf{g} , \tag{2.2}$$

$$p = \rho \mathscr{R} T , \tag{2.3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) , \tag{2.4}$$

$$\nabla \times \mathbf{B} = 0 , \tag{2.5}$$

where ρ, p, T are plasma density, pressure and temperature respectively, $\mathbf{B}$ and $\mathbf{v}$ the magnetic field and plasma velocity, $\mathbf{g}$ is solar gravity which is inversely proportional to the square of the distance from the center of the Sun, $\mathscr{R}$ the gas constant. The energy equation is not included in the equations. This means that the temperature distribution will be determined by Equation (2.3).

Generally, the tyical width of the transient configuration is much smaller than the typical scale length in the radial direction, so that the dynamical phenomena of coronal transient may be simplified to a two-dimensional process and analyzed in a plane, for example, in the (r, θ) plane of the cylindrical coordinates (r, θ, z). According to this picture, every parameter is a function of (r, θ, t) and is independent of z $(\partial/\partial z = 0)$. Furthermore, the magnetic field and the velocity may be written as

$$\mathbf{v} = (u, v, 0) , \tag{2.6}$$

$$\mathbf{B} = (B_r, B_\theta, 0) ; \tag{2.7}$$

and if so, Equations (2.1)–(2.5) may be rewritten as

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{v}{r} \frac{\partial \rho}{\partial \theta} + \rho \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} \right) = 0 , \tag{2.8}$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} \right) = -\frac{\partial p}{\partial r} - \frac{B_\theta}{4\pi r} \left(\frac{\partial r B_\theta}{\partial r} - \frac{\partial B_r}{\partial \theta} \right) - \rho \frac{GM}{r^2} , \tag{2.9}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial r} + \frac{v}{r}\,\frac{\partial v}{\partial \theta} + \frac{uv}{r}\right) = -\frac{1}{r}\,\frac{\partial p}{\partial \theta} + \frac{B_r}{4\pi r}\left(\frac{\partial r B_\theta}{\partial r} - \frac{\partial B_r}{\partial \theta}\right), \tag{2.10}$$

$$p = \rho \mathscr{R} T, \tag{2.11}$$

$$\frac{\partial B_r}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial \theta}\,(u B_\theta - v B_r), \tag{2.12}$$

$$\frac{\partial B_\theta}{\partial t} = -\frac{\partial}{\partial r}\,(u B_\theta - v B_r), \tag{2.13}$$

$$\frac{1}{r}\,\frac{\partial r B_r}{\partial r} + \frac{1}{r}\,\frac{\partial B_\theta}{\partial \theta} = 0. \tag{2.14}$$

Using condition (2.14), we introduce the magnetic potential ψ, which is defined as

$$B_r = \frac{1}{r}\,\frac{\partial \psi}{\partial \theta}, \qquad B_\theta = -\frac{\partial \psi}{\partial r}. \tag{2.15}$$

Then, the induction Equations (2.12) and (2.13) reduce to

$$\frac{\partial \psi}{\partial t} + u\,\frac{\partial \psi}{\partial r} + \frac{v}{r}\,\frac{\partial \psi}{\partial \theta} = 0; \tag{2.16}$$

and the Lorentz force in momentum Equations (2.9) and (2.10) reduces to

$$-\frac{B_\theta}{4\pi r}\left(\frac{\partial r B_\theta}{\partial r} - \frac{\partial B_r}{\partial \theta}\right) = -\frac{1}{4\pi}\,\frac{\partial \psi}{\partial r}\,L_1(\psi),$$

$$\frac{B_r}{4\pi r}\left(\frac{\partial r B_\theta}{\partial r} - \frac{\partial B_r}{\partial \theta}\right) = -\frac{1}{4\pi r}\,\frac{\partial \psi}{\partial \theta}\,L_1(\psi); \tag{2.17}$$

where the operator is defined as

$$L_1(\psi) = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\,\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\,\frac{\partial^2 \psi}{\partial \theta^2}. \tag{2.18}$$

Then, the dynamical process of the emerging magnetic region reduces to the Equations (2.8)–(2.11) and (2.16) and some initial and boundary conditions.

3. Solution of MHD Equations

The coronal transient is related closely to the eruptive prominence, and the trajectory of the prominence is given by observation. The velocity of the eruptive prominence may generally be written (see Paper I) as

$$u = u(t), \qquad v = r\Omega(t). \tag{3.1}$$

As discussed in Paper I, the eruptive prominence and the region preceding the moving prominence consist of the emerging magnetic region which moves outward and is the driving source of the coronal transient. From the observed velocity distribution of eruptive prominences (3.1), it is reasonable to assume that this velocity distribution can be applied to the whole emerging magnetic region. Substituting (3.1) into continue Equation (2.8), we have

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \Omega \frac{\partial \rho}{\partial \theta} = -\frac{\rho u}{r}. \tag{3.2}$$

The solution of (3.2) gives the density distribution as

$$\rho(r, \theta, t) = \frac{1}{r} \Theta(\hat{\xi}, \hat{\eta}), \tag{3.3}$$

where

$$\xi = r - \int u(t)\,\mathrm{d}t, \tag{3.4}$$

$$\hat{\eta} = \theta - \int \Omega(t)\,\mathrm{d}t. \tag{3.5}$$

Similarly, the induction Equation (2.16) gives the distribution of magnetic potential

$$\psi = \psi(\hat{\xi}, \hat{\eta}). \tag{3.6}$$

The distributions of $\Theta(\hat{\xi}, \hat{\eta})$ and $\psi(\hat{\xi}, \hat{\eta})$ can be determined by suitable initial conditions, and both distributions show clearly the propagation of the plasma and magnetic field.

For simplicity, we consider a stratified density and magnetic field distributions, that is, the functions Θ and ψ depends only on $\hat{\xi}$. In this case, momentum Equations (2.9) and (2.10) reduce to

$$\frac{\Theta}{r}\left(\frac{\mathrm{d}u}{\mathrm{d}t} - r\Omega^2 + \frac{GM}{r^2}\right) + \frac{1}{4\pi}\frac{\partial \psi}{\partial r} L(\psi) = -\frac{\partial p}{\partial r}, \tag{3.7}$$

$$\Theta(r\Omega' + 2u\Omega) = -\frac{\partial p}{\partial \theta}. \tag{3.8}$$

Consistency of Equations (3.7) and (3.8) requires that

$$\frac{\partial \Theta}{\partial r}(r\Omega' + 2u\Omega) + \Omega'\Theta = 0. \tag{3.9}$$

The solution of (3.9) gives the distribution of function Θ as

$$\Theta = \frac{c_1}{r + 2u\Omega/\Omega'}, \tag{3.10}$$

where c_1 is a constant. On the other hand, Θ must be a function of ξ. This condition leads to the equation

$$r_1 + 2u\Omega/\Omega' = -\int u(t)\,\mathrm{d}t\,, \tag{3.11}$$

where r_1 is a constant. From differential Equation (3.11), we have

$$\frac{\Omega}{\Omega'}\frac{\mathrm{d}u}{\mathrm{d}t} + \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Omega}{\Omega'}\right) + \frac{1}{2}\right]u = 0\,. \tag{3.12}$$

The solution of (3.12) is easily shown to be

$$u = c_2\,\frac{\Omega'}{\Omega^{3/2}}\,, \tag{3.13}$$

where c_2 is a constant. If the distribution of angular velocity is

$$\Omega = t^n \qquad (3.14$$

then, (3.13) reduces to

$$u = \frac{nc_2}{t^{1+3/2}}\,. \tag{3.15}$$

For example, if $n = -3$ the velocity distribution is given as

$$\Omega = \frac{1}{t^3}\,, \tag{3.16}$$

$$u = (-3c_2)\sqrt{t}\,, \tag{3.17}$$

where the constant c_2 is negative, and the flow is outward. The result of the radial velocity distribution (3.17) agree qualitatively with the observed trajectory of a typical eruptive prominence. The eruptive prominence is associated closely with the compressed magnetic region, the trajectory of the eruptive prominence may be considered as the trajectory of the magnetic region, which is called a 'piston' of moving plasma I. Substituting (3.16) and (3.17) into (3.4) and (3.5), we have

$$\hat{\xi} = r - R(t)\,, \qquad \hat{\eta} = \theta - \ln t\,, \tag{3.18}$$

where

$$R(t) = (-c_2)t^{3/2}\,.$$

The plasma pressure, which depends on (r, θ, t), may be determined from Equations (3.7) and (3.8). Using (3.10) and (3.11), we find that Equation (3.8) gives

$$p(r, \theta, t) = c_1\Omega'(t)\theta + p_1(r, t)\,, \tag{3.19}$$

where $p_1(r, t)$ is determined by (3.7)

$$\frac{\partial p_1(r,t)}{\partial r} = \frac{c_1}{r(\xi - r_1)}\left(u' - r\Omega^2 + \frac{GM}{r^2}\right) + \frac{1}{4\pi r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right). \tag{3.20}$$

This equation may be integrated if the function of magnetic potential ψ is given. We consider a loop-like or arcade-like configuration of magnetic field, such as

$$\psi(\hat{\xi}) = \psi_0 \exp[-(\hat{\xi} - r_2)^2/r_0^2] = \psi_0 \exp[-(r - R_0)^2/r_0^2], \tag{3.21}$$

where r_2 is a constant and

$$r_0 = R(t_0) + r_2, \qquad R_0 = R(t) + r_2. \tag{3.22}$$

Equation (3.20) then reduces to

$$\frac{\partial p_1(r,t)}{\partial r} = \frac{c_1}{r(r - R_1)}\left(u' - r\Omega^2 + \frac{GM}{r^2}\right) + $$
$$+ \frac{\psi_0^2}{\pi}\left(\frac{r - R_0}{r_0^4}\right)\left[2 - \frac{R_0}{r} - 2\left(\frac{r - R_0}{r_0}\right)^2\right]\exp[-2(r - R_0)^2/r_0^2], \tag{3.23}$$

where

$$R_1 = R(t) - r_1.$$

The solution of Equation (3.23) is of the form

$$p_1(r,t) = p_0(t) + c_1\left\{\ln\left[\frac{(r - R_1)^{k-\Omega^2}}{r}\right] + \frac{GM}{R_1^2}\frac{1}{r} + \right.$$
$$\left. + \frac{GM}{2R_1}\frac{1}{r^2}\right\} + \frac{\psi_0^2}{\pi r_0^4}\left\{\frac{r_0^2}{2}\left[\left(\frac{r - R_0}{r_0}\right)^2 - 1\right]e^{-2(r-R_0)^2} - \right.$$
$$\left. - \frac{r_0 R_0}{\sqrt{2}}\operatorname{erf}[\sqrt{2}(r - R_0)] + R_0^2\int e^{-2(r-R_0)^2}\frac{\mathrm{d}r}{r}\right\}, \tag{3.24}$$

where $p_0(t)$ is a constant of integration, and the coefficient k

$$k = \frac{GM}{R_1^3} + \frac{u'}{R_1}. \tag{3.25}$$

The above results show that the pressure gradient in the azimuthal direction balances the azimuthal component of inertial force. At the same time, the pressure gradient in the radial direction balances the solar gravity, the Lorentz force and the radial component of inertial force. The distribution of pressure is not self-similar.

It should be pointed out that we must adopt an initial time $t = t_0 > 0$ to avoid a

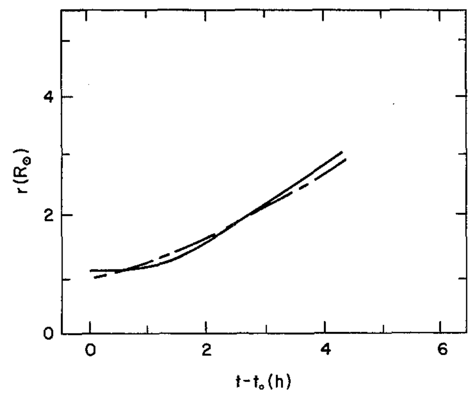

Fig. 2. Theoretical distribution of radial velocity (broken line) and its comparison with the velocity of eruptive prominence.

singularity in the above results. In this case, the initial conditions of the solution is regular.

4. Discussion of Driven Forces

In Paper I, we discussed the local features of a gas-dynamical process driven by a piston of moving plasma. As the moving piston is a region of compressed magnetic flux, its structure was discussed there. The last section gives a two-dimensional time dependent solution which may be applied to the compressed magnetic region. As an example, we discuss the density distribution. Using (3.10) and (3.11), we find the density (3.3) to be given by

$$\rho(r, t) = \frac{c_1}{r(\xi + r_1)} = \frac{c_1}{r[r - R(t) + r_1]}, \tag{4.1}$$

where we require that

$$\frac{c_1}{r - R(t) + r_1} > 0\,. \tag{4.2}$$

At initial time $t = t_0$ the density profile is

$$\rho(r, t_0) = \frac{c_1}{r[r + 2c_2 t_0^{3/2} + r_1]}, \tag{4.3}$$

where relationship (3.18) is used, the density profile includes both influences of the cross-section change (which is proportional to $1/r$) and the flow. Result (4.1) shows also the propagation property, that is, the density profile is proportional to $1/r$ for a fixed flow front ξ = constant. Similarly, the initial condition for other parameters may be discussed. It can be seen that, there are several constants in the formulas which may be adjusted to make the theoretical result agree with the observations.

The propagation of the magnetic flux region shows also clearly in relationship (3.12). If the magnetic flux region is confined in $\psi_1 \leq \psi \leq \psi_2$, and the boundaries $\psi = \psi_1$ and $\psi = \psi_2$ are tangential discontinuous surfaces, then, the trajectory of a magnetic surface $\psi = \psi$ (constant) is given by (3.21) as

$$r = R_0(t) + r_0 [\ln \psi_0]^{1/2} = R(t) + r_0 + [\ln \psi_0]^{1/2}, \tag{4.6}$$

where $\psi_1 \leq \psi \leq \psi_2$. This result means that the magnetic surface propagates with the velocity $u(t)$ along the radial direction. The width of the magnetic flux region remains constant in this special example.

Using a typical distribution of magnetic potential (3.21), the Lorentz force (2.17) may be written as

$$F(r, t) = -\frac{1}{4\pi r} \frac{\partial \psi}{\partial r} \frac{\partial}{\partial r}\left(r \frac{\partial \psi}{\partial r}\right)$$

$$= \frac{\psi_0^2}{\pi} \frac{r - R_0}{r_0^4}\left[2\left(\frac{r - R_0}{r_0}\right)^2 + \frac{R_0}{r} - 2\right] \exp\left[-\left(\frac{r - R_0}{r_0}\right)^2\right]. \tag{4.4}$$

Distribution (4.4) at the initial time $t = t_0$ reduces to

$$F(r, t_0) = \frac{\psi_0^2}{\pi} \frac{r - r_0}{r_0^4}\left[2\left(\frac{r}{r_0}\right)^2 - 4\frac{r}{r_0} + \frac{r_0}{r}\right] \exp\left[-\left(\frac{r}{r_0} - 1\right)^2\right]. \tag{4.5}$$

The profile of (4.5) is shown in Figure 3. The Lorentz force is zero at $r = r_0$, and positive in the region $0.6 < r/r_0 < 1$ and $r/r_0 < 1.85$. This result shows that the initial distribution of Lorentz force in the magnetic flux tube may be either a driven force or a resistant force. For the moving configuration, Equation (4.4) shows that

$$F(R_0, t) = 0\,, \qquad \frac{F(R_0, t)}{\partial r} = -\frac{\psi_0^2}{\pi r_0^5} < 0\,. \tag{4.7}$$

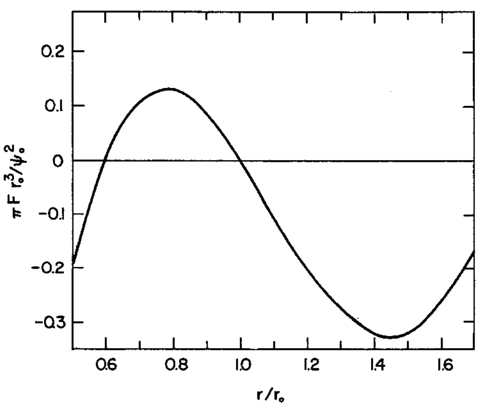

Fig. 3. Initial distribution of Lorentz force.

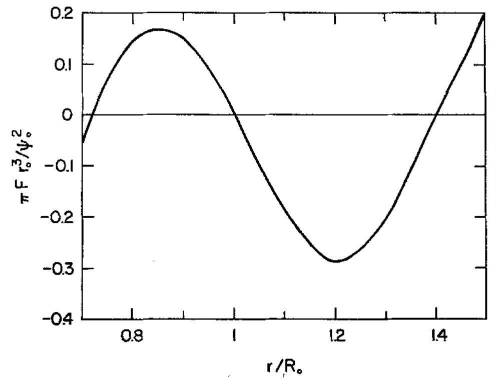

Fig. 4. The distribution of Lorentz force at $R(t) = 2r_0$.

The distribution of Lorentz force is similar to Figure 3, that is, it is positive in a region $r < R_0$ and negative in a region $r > R_0$. For example, Figure 4 gives the distribution of Lorentz force at $R_0(t) = 2r_0$. In this case, the region of positive Lorentz force increases and the region of negative Lorentz force decreases relatively. This special example shows that the Lorentz force in a flux tube need not always be positive. Both the current and the Lorentz force have a distribution in the tube. The dynamical process requires generally a complicated distributions of magnetic field and thermodynamical parameters, which are different from those assumed in simplified models. The present paper suggests that the pressure gradient is one of the important components of the driving force. Because both lifting forces against solar gravity and the acceleration of the moving plasma require some energies input, the pressure gradient and/or the Lorentz force will drive the transient. However, the pressure gradient is the only driving force in the local region if the Lorentz force is negative. The pressure gradient may be produced by the compression of moving piston. Generally, the direction and magnitude of the Lorentz force depend on the configuration of magnetic field in the transient region. There is still no clear idea suggested from observations what configuration of magnetic field should be in the transient region. This implies that we still do not know whether the Lorentz force is the driving force for coronal transient or not. The problem is open.

5. Two-Dimensional Unsteady Model for Compressed Region

Based on the idea of piston model, the piston consisted of dense plasma and strong magnetic field moves in the corona, and produces the compressed flow ahead the piston and the rarefaction flow behind the piston as shown in Figure 1. In the next three sections, we will discuss the flow field in the compressed region. As the magnetic field in the compressed region is relatively weak, we neglect the influence of magnetic field as a first step, and the flow features are described approximately by the gasdynamical theory.

Putting $B = 0$, then, Equations (2.1)–(2.5) may be reduced to the unsteady gasdynamical equations as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,, \tag{5.1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\nabla p + \rho \mathbf{g}\,, \tag{5.2}$$

$$p = \rho \mathscr{R} T\,, \tag{5.3}$$

where ρ, p, and T are plasma density, pressure, and temperature, respectively $\mathbf{v}$ is velocity $\mathbf{g}$ is solar gravitational acceleration, which is inversely proportional to the square of the distance from the solar center. The energy equation is not included here, therefore, the temperature distribution will be determined consistently.

As indicated previously, the typical width of a transient is much smaller than its typical length and height. Therefore, the transient may be considered as a planar process, and we will analyze the unsteady two-dimensional problem of gasdynamics including solar gravity. In the cylindrical coordinate system, three quantities which have the dimension of velocity can be constructed, that is,

$$\xi_1 = \frac{r}{t}\,, \qquad \eta_1 = \frac{r\theta}{t}\,, \qquad v_* = \frac{\sqrt{GM}}{r}\,, \tag{5.5}$$

where G and M are the gravitational constant and the Sun's mass, respectively. Therefore, according to the theory of dimensional analysis, the parameters of the flow field are functions of ζ_1, η_1, and t.

Equation (5.1) may be rewritten as

$$\frac{\partial \rho}{\partial t} + u\,\frac{\partial \rho}{\partial r} + \frac{v}{r}\,\frac{\partial \rho}{\partial \theta} + \rho \left(\frac{\partial u}{\partial r} + \frac{1}{r}\,\frac{\partial v}{\partial \theta} \right) = 0\,, \tag{5.6}$$

where the velocity vector is

$$\mathbf{v} = (u, v, 0)\,.$$

A two-dimensional velocity field is analyzed here. A special MHD solution with one component of velocity was discussed by Low (1982). We assume that

$$u = \frac{r}{t}\,T_3(t) \quad \text{and} \quad v = \frac{r\theta}{t}\,T_4(t)\,,$$

where $T_3(t)$ and $T_4(t)$ are known functions. The velocity components may be generally rewritten as

$$u = r\,\frac{\mathrm{d}\ln T_1(t)}{\mathrm{d}t}\,, \qquad v = r\theta\,\frac{\mathrm{d}\ln T_2(t)}{\mathrm{d}t}\,. \tag{5.7}$$

These relationships show that the two-dimensional velocity field is considered if neither u or v is not zero. For the velocity distribution (5.7), we find the characteristic equation of (5.6) to be of the form

$$\frac{\mathrm{d}t}{1} = \frac{\mathrm{d}r}{r\,\dfrac{\mathrm{d}\ln T_1}{\mathrm{d}t}} = \frac{r\,\mathrm{d}\theta}{r\theta\,\dfrac{\mathrm{d}\ln T_2}{\mathrm{d}t}} = \frac{\mathrm{d}\rho}{-\rho\left(2\,\dfrac{\mathrm{d}\ln T_1}{\mathrm{d}t} + \dfrac{\mathrm{d}\ln T_2}{\mathrm{d}r}\right)}\,. \tag{5.8}$$

Three integrals of Equation (5.8) are easily obtained as

$$\xi = \frac{r}{T_1(t)} = \text{constant}\,,$$

$$\eta = \frac{\theta}{T_2(t)} = \text{constant}\,,$$

$$\rho T_1^2 T_2 = \text{constant}\,. \tag{5.9}$$

Therefore, the solution of Equation (2.6) is given by

$$\rho(r, \theta, t) = \frac{1}{T_1^2(t)\,T_2(t)}\,f(\xi, \eta)\,, \tag{5.10}$$

where the function $f(\xi, \eta)$ is determined by the initial condition, and $T_1(t)$ and $T_2(t)$ will be given by the consistent condition.

The momentum equations may be rewritten as

$$\rho\left[\frac{r}{T_1}\,\frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} - \frac{r\theta^2}{T_2^2}\left(\frac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2 + \frac{GM}{r^2}\right] = -\frac{\partial p}{\partial r}\,, \tag{5.11}$$

$$\rho\left[\frac{r\theta}{T_2}\,\frac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \frac{2r\theta}{T_1 T_2}\,\frac{\mathrm{d}T_1}{\mathrm{d}t}\,\frac{\mathrm{d}T_2}{\mathrm{d}t}\right] = -\frac{1}{r}\,\frac{\partial p}{\partial \theta}\,. \tag{5.12}$$

Using solution (5.5), we find the consistent condition of Equations (5.11) and (5.12) to be given by

$$\begin{aligned}
&\frac{\partial f}{\partial \eta}\,\frac{1}{T_2}\left[\frac{r}{T_1}\,\frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} - \frac{r\theta^2}{T_2^2}\left(\frac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2 + \frac{GM}{r^2}\right] - \\
&\qquad -\frac{\partial f}{\partial \xi}\,\frac{r^2\theta}{T_1}\left[\frac{1}{T_2}\,\frac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \frac{2}{T_1 T_2}\,\frac{\mathrm{d}T_1}{\mathrm{d}t}\,\frac{\mathrm{d}T_2}{\mathrm{d}t}\right] = \\
&\qquad = 2fr\theta\left[\frac{1}{T_2}\,\frac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \frac{2}{T_1 T_2}\,\frac{\mathrm{d}T_1}{\mathrm{d}t}\,\frac{\mathrm{d}T_2}{\mathrm{d}t} + \frac{1}{T_2^2}\left(\frac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2\right].
\end{aligned} \tag{5.13}$$

If we assume that the function f is independent of η, then Equation (5.13) reduces to

$$\xi \frac{\mathrm{d}f}{\mathrm{d}\xi} = -2 \frac{\dfrac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \dfrac{2}{T_2} \dfrac{\mathrm{d}T_1}{\mathrm{d}t} \dfrac{\mathrm{d}T_2}{\mathrm{d}t} + \dfrac{1}{T_2}\left(\dfrac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2}{\dfrac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \dfrac{2}{T_1} \dfrac{\mathrm{d}T_1}{\mathrm{d}t} \dfrac{\mathrm{d}T_2}{\mathrm{d}t}} f . \tag{5.14}$$

The solution of (5.14) is

$$f(\xi) = \frac{c}{\xi^a} , \tag{5.15}$$

where c and a are constants. Because a is constant, this requires that

$$2 \frac{\dfrac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \dfrac{2}{T_1} \dfrac{\mathrm{d}T_1}{\mathrm{d}t} \dfrac{\mathrm{d}T_2}{\mathrm{d}r} + \dfrac{1}{T_2}\left(\dfrac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2}{\dfrac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \dfrac{2}{T_1} \dfrac{\mathrm{d}T_1}{\mathrm{d}t} \dfrac{\mathrm{d}T_2}{\mathrm{d}t}} = a \text{ (constant)} . \tag{5.16}$$

By use of (5.16), the consistent condition reduces to

$$\left(\frac{a-2}{2}\right)\left(\frac{\mathrm{d}^2 T_2}{\mathrm{d}t^2} + \frac{2}{T_1} \frac{\mathrm{d}T_1}{\mathrm{d}t} \frac{\mathrm{d}T_2}{\mathrm{d}t}\right) = \frac{1}{T_2}\left(\frac{\mathrm{d}T_2}{\mathrm{d}t}\right)^2 . \tag{5.17}$$

Then, the density distribution is given as

$$\rho(r, t) = \frac{c}{T_1^2 T_2 \xi^a} = \frac{C T_1^{a-2}}{r^a T_2} . \tag{5.18}$$

If we substitute the solution (5.15) in the momentum Equation (5.11) or (5.12), the pressure distribution is obtained as

$$p(r, \theta, t) = p_0(t) + \frac{C T_1^{a-2}}{T_2} \left\{ \left[\frac{1}{T_1} \frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} - \frac{\theta^2}{T_2^2} \left(\frac{\mathrm{d}T_2}{\mathrm{d}t} \right)^2 \right] \frac{r^{2-a}}{a-2} + \frac{1}{1+a} \frac{GM}{r^{1+a}} \right\} ,$$
$$\text{if} \quad a \neq 2, -1$$

$$p(r, \theta, t) = p_0(t) + \frac{c}{T_2} \left\{ \left(\frac{1}{T_1} \frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} - \frac{\theta^2}{T_2} \left(\frac{\mathrm{d}T_2}{\mathrm{d}t} \right)^2 \right] \ln r + \frac{GM}{3r^2} \right\} ,$$
$$\text{if} \quad a = 2 \quad (5.19)$$

$$p(r, \theta, t) = p_0(t) + \frac{c}{T_1^3 T_2} \left\{ -\frac{r^3}{3} \left[\frac{1}{T_1} \frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} + \frac{\theta^2}{T_2} \left(\frac{\mathrm{d}T_2}{\mathrm{d}t} \right)^2 \right] + GM \ln r \right\} ,$$
$$\text{if} \quad a = -1 ;$$

where p_0 is a function of t. The temperature may be given from the equation (5.3) of state.

These results show that the density distribution obeys some similarity relations but the distributions of pressure and temperature do not. The consistent conditions has reduced to Equation (5.17), which contains two functions $T_1(t)$ and $T_2(t)$. These functions need to be determined. In general, we can choose the density profile parameter and a radial velocity profile $T_1(t)$, then solve the profile function of azimuthal velocity $T_2(t)$, and vice versa. According to these relationships, the entropy of plasma is changed with time and space. This means that there is a heating source input in the fluid.

6. Compressed Flow

In the piston problem of gasdynamics, there is a uniform compressed flow ahead of the piston if the cross section of the flowing tube is constant and applied gravity is excluded. The gas density will decrease if the cross section increases, which is the case for transient flow in the corona. That is, the density of compressed flow preceding the moving piston will gradually decrease because the cross section of the transient flow increases as the distance r increases. According to (5.18), the density is inversely proportional to r^a in the region between the moving piston and the shock front. This result may be compared with observations. From the viewpoint of energy, the plasma will dissipate its energy to overcome the resistance of applied gravity. The influence of applied gravity may decrease both the kinetic energy and the internal energy of the plasma (Hu, 1982a). However, the results in this section show that the kinetic energy will increase if x/t increases. The reason for the different conclusions is that in the present paper, the entropy has to be increased, and the heat source relating to the increasing entropy may overcome the solar gravity and even accelerate the plasma. But the physical mechanism of the heat source suggested in the present paper is not very clear. One probability is the dissipation of the magnetic energy in the small-scale structure (see, for example, Hu, 1982b). The influence of gravity on compressed flow discussed elsewhere (Hu, 1983d, e).

Because the solution given above is one of the special solutions, the special boundary conditions are required. Applied to the piston problem associated with a transient, the piston velocity should be two-dimensional and should match the profile of (5.7), because the gas velocity mthe same as the piston velocity at the surface of the moving piston. On the other hand, the discontinue condition of the shock wave should be satisfied at the interface of the compressed flow region and the quiet corona.

The conservation conditions for the shock wave may be written as

$$\rho(D - v_n) = \rho_c D\,, \tag{6.1}$$

$$\rho(D - v_n)^2 + p = \rho_c D^2 + p_c\,, \tag{6.2}$$

$$v_\tau = 0\,, \tag{6.3}$$

$$\tfrac{1}{2}(D - v_n)^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{D^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_\tau}{\rho_\tau}\,, \tag{6.4}$$

and at the shock front

$$r = r_s(t), \qquad \theta = \theta_s(t). \tag{6.5}$$

The subscription s denotes the value of the parameter at the quiet corona very near the shock front. The subscriptions n and τ denote, respectively, the normal and tangential components at the local position of shock front; for example, we have

$$\begin{aligned} v_n &= u \cos\alpha + v \sin\alpha, \\ v_\tau &= -u \sin\alpha + v \cos\alpha, \end{aligned} \tag{6.6}$$

where α is the angle between the radial direction and the normal direction at the local position of the shock front. Equations (6.1)–(6.4) depend on time t. We have four adjusting parameters: shock wave velocity $D(r, \theta, t)$, two functions of shock front (6.5) and one of the velocity profile functions $T_1(t)$ and $T_2(t)$. In principle, the jump conditions of the shock wave may be satisfied.

By the use of (6.1), the shock velocity D can be determined from Equation (6.2) as

$$D = \sqrt{\frac{\rho(p - p_\tau)}{\rho_\tau(\rho - \rho_\tau)}}. \tag{6.7}$$

Substituting (6.6) and (6.7) into (6.1), we find the normal component of velocity v_n to be given by

$$u \cos\alpha + v \sin\alpha = \sqrt{\frac{(\rho - \rho_\tau)(p - p_\tau)}{\rho\rho_\tau}}; \tag{6.8}$$

and the relationship (6.3) requires that

$$u \sin\alpha = v \cos\alpha. \tag{6.9}$$

Relationships (6.8) and (6.9) can be satisfied if we adjust in inclination angle of the shock wave α, or equivalently, the profile functions of the shock front (6.5). Finally, the conservation condition of energy, (6.3), reduces to

$$\left(\frac{\gamma+1}{\gamma-1}\, p_\tau\right)\rho + \rho p - \left(\frac{\gamma+1}{\gamma-1}\,\rho_\tau\right)p = \rho_\tau p_\tau. \tag{6.10}$$

Equation (3.10) may be rewritten as

$$\begin{aligned} \frac{1}{T_1}\frac{d^2 T_1}{dt^2} - \frac{\theta^2}{T_2^2}\left(\frac{dT_2}{dt}\right)^2 &= S_1(r)\,\frac{T_1^{2-a}\,T_2}{c} \times \\ &\times \left[\frac{\rho_\tau P_\tau}{\dfrac{c}{T_1^2 T_2 \xi^a} - \dfrac{\gamma+1}{\gamma-1}\,\rho_\tau} - \frac{c p_\tau}{c\,\dfrac{\gamma-1}{\gamma+1} - T_1^2 T_2 \xi^a \rho_\tau} - p_0(t)\right] + GMS_2(r), \end{aligned} \tag{6.11}$$

where

$$S_1(r) = \begin{cases} \dfrac{r^{2-a}}{a-2}, & \text{if} \quad a \neq 2, \\ \ln r, & a = 2; \end{cases} \tag{6.12}$$

and

$$S_2(r) = \begin{cases} \dfrac{1}{1+a} \dfrac{1}{r^{1+a}}, & \text{if} \quad a \neq -1, \\ \ln r, & \text{if} \quad a = -1. \end{cases} \tag{6.13}$$

Equation (6.11) is a differential equation for functions $T_1(t)$ and $T_2(t)$. Combining this equation with the equation of consistent condition (5.17), we can determine both functions $T_1(t)$ and $T_2(t)$. Therefore, the conservation relationships of the shock wave are satisfied, and the configuration and strength of the shock wave may be given.

7. An Example for Compressed Region

We discuss a simple case, and assume that

$$a = 2. \tag{7.1}$$

Then, the consistent condition (5.17) requires that

$$T_2 = \text{constant}. \tag{7.2}$$

This result implies that the thermodynamical parameters are independent of θ. In this case, shock velocity condition (6.9) requires that

$$\sin \alpha = 0,$$

or we have

$$\alpha = 0. \tag{7.3}$$

This result shows that the shock front is a cylindrical surface, and may be expressed as

$$r = r_s(t). \tag{7.4}$$

Another condition of shock velocity (6.8) requires that

$$r_s \frac{\mathrm{d} \ln T_1}{\mathrm{d}t} = \sqrt{\frac{(\rho - \rho_\tau)(p - p_\tau)}{\rho p_\tau}} \quad \text{at} \quad r = r_s(t). \tag{7.5}$$

On the other hand, the normal shock velocity is given by (6.6) as

$$D = \frac{\mathrm{d}r_s}{\mathrm{d}t} = \sqrt{\frac{\rho(p - p_\tau)}{\rho_\tau(\rho - \rho_\tau)}} \quad \text{at} \quad r = r_s(t). \tag{7.6}$$

Both Equations (7.5) and (7.6) may give the functions of the velocity profile $T_1(t)$ and the shock profile $r_s(t)$, if the thermodynamical parameters ρ_τ and p_τ in the quiet corona are given. This result means that the shock velocity is closely related with the flow velocity of the plasma. There is another parameter $p_0(t)$ in the relation of pressure distribution (5.19). Function $p_0(t)$ is determined by Equation (6.11) when $T_1(t)$ and the parameter in the quiet corona are given. Then, the flow field and shock wave are solved. The physical meaning is that both the shock velocity and the plasma velocity are related with the thermodynamical parameters of the plasma.

The distributions of density and pressure are given by (5.18) and (5.19), respectively, as

$$\rho = \rho_0 \left(\frac{r_0}{r}\right)^2 \tag{7.7}$$

and

$$p = p_0(t) + \rho_0 r_0^2 \left[\frac{1}{T_1} \frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} \ln r + \frac{GM}{3r^2}\right], \tag{7.8}$$

where the constant c in Equation (5.18) is

$$c = \rho_0 r_0^2 / T_2 .$$

Based on the hypothesis in Paper I, the front of the shock wave ahead to the front of the coronal transient, which propagates with nearly constant velocity or slight acceleration, as shown by observations. For simplicity, we consider the case of constant shock velocity, that is:

$$D = \frac{\mathrm{d}r_s}{\mathrm{d}t} = D_0 \,(\text{constant}), \tag{7.9}$$

or

$$r_s = D_0 T + r_{s0} . \tag{7.10}$$

Substituting (7.10) into (6.16), we obtain the equation of $T_1(t)$ as

$$\frac{\mathrm{d}\ln T_1}{\mathrm{d}t} = \frac{D_0}{D_0 t + r_{s0}} \left[1 - \frac{\rho_c (D_0 t + r_{0s})^2}{\rho_0 r_0^2}\right]. \tag{7.11}$$

This shows that the radial velocity of the plasma decreases gradually for constant r/t. The solution of Equation (7.11) gives the profile function of velocity $T_1(t)$ as

$$T_1(t) = T_0 (D_0 t + r_{s0}) \exp\left[-\frac{1}{\rho_0 r_0^2} \int^{x = D_0 t + r_{0s}} x \rho_c(x)\,\mathrm{d}x\right]. \tag{7.13}$$

From these results, relation (6.17) gives the function $p_0(t)$ as

$$p_0(t) = \rho_\tau D_0^2 \left(1 - \frac{\rho_\tau r_s^2}{\rho_0 r_0^2}\right) + p_\tau -$$

$$- \rho_0 r_0^2 \left[\frac{1}{T_1} \frac{\mathrm{d}^2 T_1}{\mathrm{d}t^2} \ln r + \frac{GM}{3r^2}\right] \quad \text{when} \quad r = D_0 t + r_{s0} . \tag{7.14}$$

Therefore, the flow field of the compressed plasma and the relationship of the shock wave are solved completely for this simple case. The results of this example show that the plasma density decreases with increasing distance r and is inversely proportional to r^4 for this special case, the plasma velocity decreases gradually for fixed x/t if the velocity of the shock front remains constant, and the pressure gradient in the radial direction balances with the solar gravity and the inertial force. The quantitative results may be easily obtained if the parameters in the quiet corona are given quantitatively.

Observations show that the configuration of the transient front is generally two-dimensional, depending on both r and θ. Based on the above analysis, the shock front is really two-dimensional and unsteady, so the density increment which corresponds to the bright feature of the transient has a two-dimensional configuration, although the solution of density between the piston and shock front depends only on r. The observations often show that the propagation velocities of many coronal transients are smaller than the local sonic velocity, and therefore the bright feature of the transient cannot correspond to the shock front. As discussed in Paper I, the shock wave is located at the crossing points of the characteristic lines. The slower the piston velocity or acceleration, the larger the distance between the piston and the shock front. On the other hand, the density of the compressed plasma will decrease from the piston to the shock wave under the influence of solar gravity. In this case, the strength of the shock wave is relatively weak, and the high density region is confined near the piston, whose velocity is smaller than the local sonic velocity. Therefore, the bright feature of this kind of transient will propagate with a velocity smaller than the local sonic velocity.

8. Discussion

Usually, the local approach is simple and easy to give a clear physical picture. We suggested two local solutions in Paper I, they describe the sketch of the piston model for coronal transient. Many observations show that the coronal transient is, at least, two-dimensional phenomenon. Therefore, in the present paper, we suggest the two-dimensional analytical solutions in the finite regions, these solutions show the dynamical mechanism of the transient. It should be noted that the solutions in this paper are limited in the finite region, otherwise, there will be singularity at infinite. Similar conclusions may be found elsewhere, for example, the Riemann flow field of piston problem is confined in the region between the piston and the shock wave. This is the reason why we discuss the solutions in the finite regions. From the view point of mathematics, the physical process is described not only by the equations, but also the boundary conditions. Same equation may give obviously different physical picture if the boundary conditions are different. In the present paper, the solutions are checked by the observations, for examples, the trajectory of the prominence and the shock front conditions. On the other hand, it will be interesting to study the unsteady and two-dimensional computations, which may give the dynamical evolution of the complete picture.

The coronal transients have different topographies: loops, clouds, filled bottles, rays, material injected into streamers and others. The observations show that most events of coronal transients associate with eruptive prominences and/or solar flares (Munro *et al.*, 1979). In the present paper and Paper I, we suggest that the transients are the responses of dense plasma ejecting into the active region, and analyze especially the connection between the transient and the eruptive prominence because many evidences show clearly the connection. However, there may has the ejecting dense plasma produced by the solar flare, and then, the similar mechanism. As the solar flare depends closely with the magnetic field in the active region, there may has connections between the topography of transient and the configuration of the magnetic field. It is easy to form the loops, clouds and bottles if the dense plasma ejecting into the closed magnetic field, as shown in Figure 5. The topographies of rays and the material injecting into streamers may be explained by the dense plasma ejecting into the open magnetic field, as shown in Figure 6. Both mechanism are suggested (Hu, 1983b, c). As the local magnetic fields have complex configurations, and hence, the coronal transients may be manifested in different topographies. However, the basic mechanism may be similar, that is, dense plasma ejects into corona where has some distributions of magnetic fields.

Similar to the solar flare, the energy source is one of the important problems for transient event. In the Paper I, we estimate the kinetic energy of eruptive prominence, and show that the kinetic energy may has same magnitude order as the typical event of the coronal transient, but not always has. On the other hand, the eruptive prominence

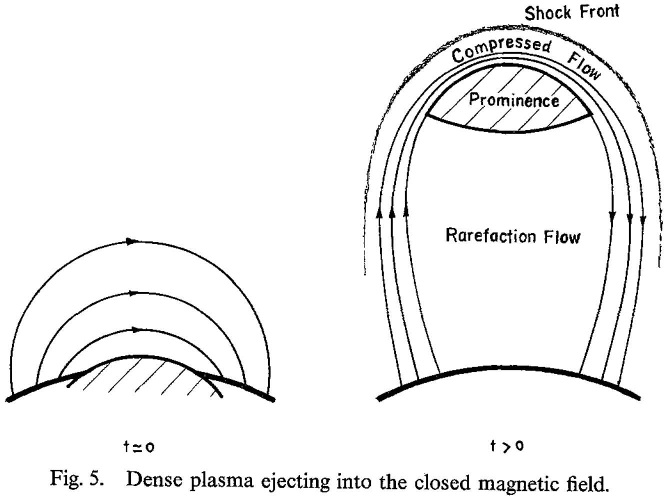

Fig. 5. Dense plasma ejecting into the closed magnetic field.

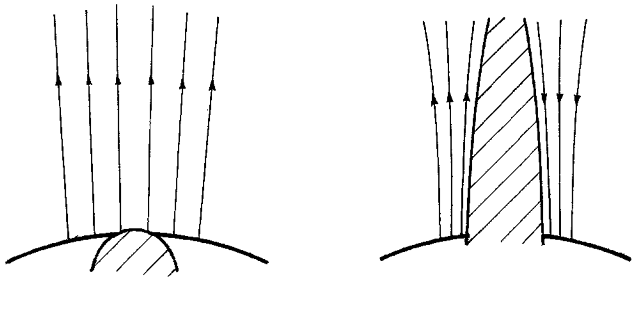

Fig. 6. Dense plasma ejecting into the open magnetic field.

will remains in the low corona, and the compressed magnetic region may move outward to the outer corona. This means that the piston will separate. In this case, the magnetic region, which is driven by the eruptive prominence in the low corona, may be accelerated continuously by the magnetic force, and the magnetic energy converts into kinetic or internal energy to driven the transient. Therefore, both the kinetic energy of the eruptive prominence and the magnetic energy in the magnetic flux region may be the energy source of the coronal transient, and the total energy of both can larger than 10^{30} ergs, which is the typical energy of coronal transient. In this case, the eruptive prominence is, in some meanings, the trigger for the conversion of the magnetic energy.

The results of the present paper may have other applications. It is known that the consistent solutions of unsteady and two-dimensional equations of magnetohydrodynamics and gasdynamics with applied gravity have extensive applications in the astrophysical processes, and they are still the difficult subjects in theory. Of course, on the other hand, the special solution itself has many limitations when applied to the special astrophysical process, because it requires to satisfy some special conditions. Therefore, although these solutions may shed light on the mechanism of the coronal transient, we must apply it carefully and avoid introducing nonphysical conditions.

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